Boolean Algebraic Structure of Finite Soft Sets

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Abstract

This paper investigates the matrix representation of soft sets over finite universes and finite parameter spaces. Each soft set is represented uniquely by a binary matrix whose entries indicate the membership or non-membership of objects with respect to the given parameters. The matrix forms of fundamental soft set operations, including union, intersection, and complement, are established through entrywise Boolean operations. It is proved that the correspondence between soft sets and binary matrices is bijective. Furthermore, several algebraic properties, such as idempotent, commutative, associative, absorption, distributive, identity, and complementation laws, are derived for the corresponding matrix operations. Finally, it is shown that the collection of matrix representations of soft sets, equipped with suitable operations and complements, forms a Boolean algebra. These results provide an algebraic framework for representing and manipulating finite soft sets using binary matrices.

Keywords: Soft sets; Matrix representation; Binary matrices; Finite parameter space; Boolean algebra; Soft set operations; Matrix operations; Complement of soft sets.

1. Introduction

Uncertainty and incomplete information are common features of many real-world problems, particularly in decision-making, data analysis, medical diagnosis, pattern recognition, and information systems. Classical mathematical tools often require precise information, whereas practical problems frequently involve parameters whose values cannot be described adequately by conventional sets. To address such situations, Molodtsov introduced the concept of a soft set as a flexible mathematical framework for dealing with parameterized information. Since its introduction, soft set theory has been studied extensively and has been extended to several related structures and applications.

In a soft set, the relationship between objects and parameters is represented through a parameterized family of subsets of the universe. When both the universe and the parameter set are finite, this relationship can be expressed naturally in the form of a binary matrix. Such a representation provides a convenient connection between soft set theory and matrix-based computation. In particular, the rows of the matrix represent the elements of the universe, while the columns correspond to the associated parameters. The entries indicate whether a particular element satisfies a given parameter.

Matrix representation is useful not only for storing soft set information but also for performing soft set operations systematically. Union, intersection, and complement can be expressed through elementary operations on binary matrices. This representation also facilitates computational implementation, since finite soft sets can be handled using standard matrix-processing techniques.

The present study develops the algebraic structure associated with the matrix representation of finite soft sets. The correspondence between soft sets and binary matrices is established, and the matrix forms of fundamental soft set operations are investigated. Several algebraic laws are subsequently derived for these operations. In particular, the relationships among idempotency, commutativity, associativity, absorption, distributivity, identity elements, and complementation are examined. Finally, these properties are used to establish the Boolean algebraic structure of the matrix representation of soft sets.

The results provide a systematic mathematical foundation for representing finite soft sets through binary matrices and may be useful in computational applications where parameterized information needs to be stored, processed, and analyzed efficiently.

1.Preliminary

Soft set theory has been initiated by Molodtsov[8] in the year 1999 in order to represent vagueness in real life problems. Following this, many researchers[1,2,3,4,5,6,7,10,11,12,13, 17, 18] concentrated their work in the application of soft sets. One such application is soft topology introduced by Muhammed Shabir and Munazza Naz[9]. Recently several topological concepts have been extended to soft topological spaces by the researchers in soft topology.
If (X, τ) is a topological space, A and B are subsets of X then A is near to B if *Int*A = *Int*B. Throughout this paper, X is a non empty set and E is a parameter space. By a soft subset F of

X with parameter space E, we mean a function F: $E \rightarrow 2^X$. Let S(X, E) be the collection of all soft subsets of X with parameter space E.

We use the following notations in soft topological spaces. If F and G are soft sets in S(X, E) then

$F \sqsubseteq G$ means F is a soft subset of G,

$F \sqsupseteq G$ means F is a soft superset of G,

$F \sqcap G$ means soft intersection between F and G,

$F \sqcup G$ means soft union between F and G,

$F \boxminus G$ means soft difference G from F.

Definition 1.1: Let τ be a sub collection of S(X, E). Then τ is said to be a soft topology[9] on X if

(i) $\varnothing_E$ and X_E belong to τ.

(ii) τ is closed under finite soft intersection.

(iii) τ is closed under arbitrary soft union.

If τ is a soft topology on X then the triplet (X, τ, E) is called a soft topological space over (X, E) and τ is a soft topology over (X, E). The members of τ are called the soft open sets in (X, τ, E). The soft complement of a soft open set is soft closed. The soft interior and soft closure of a soft set can be defined in the usual way. If $F \in$ S(X, E) then *softInt* F = the soft interior of F in (X, τ, E) and *softCl* F = the soft closure of F in (X, τ, E).

The next lemma, due to Shabir Hussain and Bashir Ahmad[14], gives the parametrized family of topologies induced by a soft topology.

Lemma 1.2: Let (X, τ, E) be a soft topological space. Then for each $e \in E$, the collection of sub sets F(e) of X, $F \in \tau$ is a topology on X, denoted by τ_e.

The family $\{\tau_e : e \in E\}$ is called a parametrized family of topologies induced by the soft topology τ. The following operators on soft sets are useful in sequel.

$\lambda(F) = softCl(softInt F)$.

$\mu(F) = softInt(softCl F)$.

$\gamma(F) = softInt(softCl(softInt F))$.

$\eta(F) = softCl(softInt(softCl F))$.

Definition 1.3:[6,7] A soft set F in a soft topological space (X, τ, E) is called

(i) soft regular open if $F = \mu(F)$ and soft regular closed if $F = \lambda(F)$,

(ii) soft α-open if $F \sqsubseteq \gamma(F)$ and soft α-closed if $\eta(F) \sqsubseteq F$,

(iii)soft semi-open if $F \sqsubseteq \lambda(F)$ and soft semi-closed if $\mu(F) \sqsubseteq F$,

(iv) soft pre-open if $F \sqsubseteq \mu(F)$ and soft pre-closed if $\lambda(F) \sqsubseteq F$,

(v) soft β-open if $F \sqsubseteq \eta(F)$ and soft β-closed if $\gamma(F) \sqsubseteq F$,

(vi) soft b-open if $F \sqsubseteq \mu(F) \sqcup \lambda(F)$ and soft b-closed if $\mu(F) \sqcap \lambda(F) \sqsubseteq F$,

(vii) soft *b-open if $F \sqsubseteq \mu(F) \sqcap \lambda(F)$ and soft *b-closed if $\mu(F) \sqcup \lambda(F) \sqsubseteq F$,

(viii) soft $b^{\#}$-open if $F = \mu(F) \sqcup \lambda(F)$ and soft $b^{\#}$-closed if $\mu(F) \sqcap \lambda(F) = F$,

(ix) a soft q-set if $\mu(F) \sqsubseteq \lambda(F)$,

(x) a soft p-set if $\lambda(F) \sqsubseteq \mu(F)$ and

(xi) a soft Q-set if $\mu(F) = \lambda(F)$.

Definition 1.4: Let F and G $\in$ S(X, E) and let (X, τ, E) be a soft topological space. F is soft near to G [13] if *softInt* F = *softInt* G.

2 Matrix Representation of soft sets

Most of the sets and associated space of parameter elements in real life applications are finite sets. For example in any hospital there are only a finite number of patients over a period and the parameters related to their medical history constitute a finite set. Therefore we assume that X is a finite set of discourse with finite parameter space E. Let $X = \{x_1,x_2,x_3,\ldots,x_m\}$ and $E=\{e_1,e_2,e_3,\ldots,e_n\}$. If $\tilde{F} \in \tilde{S}(X, E)$ then it can be identified by a matrix $Mat\,(\tilde{F}) = [f_{ij}]$ with m rows and n columns where $f_{ij} = 1$ if $x_i \in \tilde{F}(e_j)$ and $= 0$ if $x_i \notin \tilde{F}(e_j)$.

Theorem 2.1:

Let $\tilde{F}$ and $\tilde{G}$ be any two soft sets in $\tilde{S}$(X, E) with $Mat\ (\tilde{F}) = [f_{ij}]$ and $Mat\ (\tilde{G}) = [f_{ij}]$. Then the following results hold.

(i) $Mat\ (\tilde{F}), Mat\ (\tilde{G}) \in M(\{0,1\}, m\times n)$.

(ii) $Mat\ (\tilde{F}) \oplus Mat\ (\tilde{G}) = [f_{ij} \oplus g_{ij}]$.

(iii) $Mat\ (\tilde{F}) \odot Mat\ (\tilde{G}) = [f_{ij} \odot g_{ij}]$.

(iv) $(Mat\ (\tilde{F}))' = Mat\ (\tilde{X} \boxminus \tilde{F})$.

Proof:

Let $\tilde{F}$ and $\tilde{G}$ be any two soft sets in $\tilde{S}$(X, E).

$\Rightarrow Mat\ (\tilde{F}) = [f_{ij}]$ and $Mat\ (\tilde{G}) = [g_{ij}]$ (where $f_{ij} = 1$ if $x_i \in \tilde{F}(e_j)$ and $= 0$ if $x_i \notin \tilde{F}(e_j)$, $g_{ij} = 1$ if $x_i \in \tilde{G}(e_j)$ and $= 0$ if $x_i \notin \tilde{G}(e_j)$)

$\Rightarrow f_{ij} = 0$ or 1, $g_{ij} = 0$ or 1

$\Rightarrow [f_{ij}], [g_{ij}] \in M(\{0,1\}, m\times n)$

$\Rightarrow Mat\ (\tilde{F}), Mat\ (\tilde{G}) \in M(\{0,1\}, m\times n)$. This proves (i).

Let $Mat\ (\tilde{F}) = [f_{ij}]$, $Mat\ (\tilde{G}) = [g_{ij}]$ and $Mat\ (\tilde{F}) \oplus Mat\ (\tilde{G}) = [x_{ij}]$.

x_{ij} = The $(i,j)^{th}$ entry in $Mat\ (\tilde{F}) \oplus Mat\ (\tilde{G})$

= The $(i,j)^{th}$ entry in $[f_{ij}] \oplus [g_{ij}] = f_{ij} \oplus g_{ij}$

= The $(i,j)^{th}$ entry in $[f_{ij} \oplus g_{ij}]$

The $(i,j)^{th}$ entry in $[f_{ij} \oplus g_{ij}] = f_{ij} \oplus g_{ij}$ = The $(i,j)^{th}$ entry in $[f_{ij}] \oplus [g_{ij}]$

= The $(i,j)^{th}$ entry in $Mat\ (\tilde{F}) \oplus Mat\ (\tilde{G})$

$= x_{ij}$. This proves (ii).

Let $Mat\ (\tilde{F}) \odot Mat\ (\tilde{G}) = [y_{ij}]$

y_{ij} = The $(i,j)^{th}$ entry in $Mat\ (\tilde{F}) \odot Mat\ (\tilde{G})$

= The $(i,j)^{th}$ entry in $[f_{ij}] \odot [g_{ij}]$

$= f_{ij} \odot g_{ij}$ = The $(i,j)^{th}$ entry in $[f_{ij} \odot g_{ij}]$

The $(i,j)^{th}$ entry in $[f_{ij} \odot g_{ij}] = f_{ij} \odot g_{ij}$ = The $(i,j)^{th}$ entry in $[f_{ij}] \odot [g_{ij}]$

= The $(i,j)^{th}$ entry in $Mat\ (\tilde{F}) \odot Mat\ (\tilde{G})$

$= y_{ij}$. This proves (iii).

The $(i,j)^{th}$ entry in $Mat\ (\tilde{X} \boxminus \tilde{F})$ is 1 $\Leftrightarrow x_i \in (\tilde{X} \boxminus \tilde{F})\ (e_j)$

$\Leftrightarrow x_i \notin \tilde{F}(e_j)$

$\Leftrightarrow$ The $(i,j)^{th}$ entry in $Mat\ (\tilde{F})$ is 0

The $(i,j)^{th}$ entry in $Mat\ (\tilde{X} \boxminus \tilde{F})$ is 0 $\Leftrightarrow x_i \notin (\tilde{X} \boxminus \tilde{F})(e_j)$

$\Leftrightarrow x_i \in \tilde{F}(e_j)$

$\Leftrightarrow$ The $(i,j)^{th}$ entry in $Mat\ (\tilde{F})$ is 1. This proves (iv).

Theorem 2.2:

If $[h_{ij}] \in M(\{0,1\}, m\times n)$ then there is a unique soft set $\widetilde{H}$ in $\tilde{S}(X, E)$ such that $Mat\ (\widetilde{H}) = [h_{ij}]$.

Proof:

Let $[h_{ij}]$ be a matrix over {0,1}. Let $\widetilde{H}$ be defined as $x_i \in \widetilde{H}(e_j)$ if $h_{ij} = 1$ and $x_i \notin \widetilde{H}(e_j)$ if $h_{ij} = 0$. Clearly $Mat\ (\widetilde{H}) = [h_{ij}]$. Suppose $\widetilde{K}$ is any soft set with $Mat\ (\widetilde{K}) = [h_{ij}]$. Then $x_i \in \widetilde{K}(e_j)$ $\Leftrightarrow h_{ij} = 1 \Leftrightarrow x_i \in \widetilde{H}(e_j)$. Therefore $\widetilde{H} = \widetilde{K}$. This proves the theorem.

Theorem 2.3:

(i) The association $\tilde{F} \to Mat\ (\tilde{F})$ is a bijection from $\tilde{S}(X, E)$ to $M(\{0,1\}, m\times n)$

(ii) $M(\{0,1\}, m\times n) = \{Mat\ (\tilde{F}) : \tilde{F} \in \tilde{S}(X, E)\} = M(\tilde{S}(X, E))$.

(iii) $Mat\ (\widetilde{\emptyset}) = [0_{ij}]$

(iv) $Mat\ (\tilde{X}) = [1_{ij}]$

Proof:

The assertions (i) and (ii) follows from Theorem 2.1 and Theorem 2.2. The proof for (iii) and (iv) is obvious.

Theorem 2.4:

For $\tilde{F}, \tilde{G}, \widetilde{H} \in \tilde{S}(X, E)$, the following properties hold.

(i) Idempotent laws: $Mat\ (\tilde{F}) \oplus Mat\ (\tilde{F}) = Mat\ (\tilde{F}) = Mat\ (\tilde{F}) \odot Mat\ (\tilde{F})$.

(ii) Commutative laws: $Mat\ (\tilde{F}) \oplus Mat\ (\tilde{G}) = Mat\ (\tilde{G}) \oplus Mat\ (\tilde{F})$,

$$Mat\ (\tilde{F}) \odot Mat\ (\tilde{G}) = Mat\ (\tilde{G}) \odot Mat\ (\tilde{F}).$$

(iii) Associative laws:

$$(Mat\ (\tilde{F}) \oplus Mat\ (\tilde{G})) \oplus Mat\ (\widetilde{H}) = Mat\ (\tilde{F}) \oplus (Mat\ (\tilde{G}) \oplus Mat\ (\widetilde{H})),$$

$$(Mat\ (\tilde{F}) \odot Mat\ (\tilde{G})) \odot Mat\ (\widetilde{H}) = Mat\ (\tilde{F}) \odot (Mat\ (\tilde{G}) \odot Mat\ (\widetilde{H}))$$

(iv) Existence of universal bounds:

$Mat\ (\tilde{F}) \oplus Mat\ (\widetilde{\emptyset}) = Mat\ (\widetilde{\emptyset}) \oplus Mat\ (\tilde{F}) = Mat\ (\tilde{F})$,

$Mat\ (\tilde{F}) \odot Mat\ (\widetilde{\emptyset}) = Mat\ (\widetilde{\emptyset}) \odot Mat\ (\tilde{F}) = Mat\ (\widetilde{\emptyset})$,

$Mat\ (\tilde{F}) \oplus Mat\ (\tilde{X}) = Mat\ (\tilde{X}) \oplus Mat\ (\tilde{F}) = Mat\ (\tilde{X})$,

$Mat\ (\tilde{F}) \odot Mat\ (\tilde{X}) = Mat\ (\tilde{X}) \odot Mat\ (\tilde{F}) = Mat\ (\tilde{F})$.

(v) Absorption laws: $Mat\ (\tilde{F}) \oplus (Mat\ (\tilde{F}) \odot Mat\ (\tilde{G})) = Mat\ (\tilde{F})$.

$$Mat\ (\tilde{F}) \odot (Mat\ (\tilde{F}) \oplus Mat\ (\tilde{G})) = Mat\ (\tilde{F}).$$

(vi) Mutually distributive laws:

$Mat\,(\tilde{F}) \oplus (Mat\,(\tilde{G}) \odot Mat\,(\tilde{H})) = (Mat\,(\tilde{F}) \oplus Mat\,(\tilde{G})) \odot (Mat\,(\tilde{F}) \oplus Mat\,(\tilde{H}))$,

$Mat\,(\tilde{F}) \odot (Mat\,(\tilde{G}) \oplus Mat\,(\tilde{H})) = (Mat\,(\tilde{F}) \odot Mat\,(\tilde{G})) \oplus (Mat\,(\tilde{F}) \odot Mat\,(\tilde{H}))$.

(vii) Complementation laws:

$Mat\,(\tilde{F}) \oplus (Mat\,(\tilde{F}))' = (Mat\,(\tilde{F}))' \oplus Mat\,(\tilde{F}) = Mat\,(\tilde{X})$,

$Mat\,(\tilde{F}) \odot (Mat\,(\tilde{F}))' = (Mat\,(\tilde{F}))' \odot Mat\,(\tilde{F}) = Mat\,(\tilde{\emptyset})$.

Proof:

Let $Mat\,(\tilde{F}) = [f_{ij}]$, $Mat\,(\tilde{G}) = [g_{ij}]$, $Mat\,(\tilde{H}) = [h_{ij}]$. By using Theorem 2.1.6 we have

$Mat\,(\tilde{F}) \oplus Mat\,(\tilde{F}) = [f_{ij}] \oplus [f_{ij}] = [f_{ij}] = Mat\,(\tilde{F})$ and

$Mat\,(\tilde{F}) \odot Mat\,(\tilde{F}) = [f_{ij}] \odot [f_{ij}] = [f_{ij}] = Mat\,(\tilde{F})$. This proves (i).

$Mat\,(\tilde{F}) \oplus Mat\,(\tilde{G}) = [f_{ij}] \oplus [g_{ij}] = [g_{ij}] \oplus [f_{ij}] = Mat\,(\tilde{G}) \oplus Mat\,(\tilde{F})$ and

$Mat\,(\tilde{F}) \odot Mat\,(\tilde{G}) = [f_{ij}] \odot [g_{ij}] = [g_{ij}] \odot [f_{ij}] = Mat\,(\tilde{G}) \odot Mat\,(\tilde{F})$. This proves (ii).

$$\begin{aligned}(Mat\,(\tilde{F}) \oplus Mat\,(\tilde{G})) \oplus Mat\,(\tilde{H}) &= ([f_{ij}] \oplus [g_{ij}]) \oplus [h_{ij}] \\ &= [f_{ij}] \oplus ([g_{ij}] \oplus [h_{ij}]) \\ &= Mat\,(\tilde{F}) \oplus (Mat\,(\tilde{G}) \oplus Mat\,(\tilde{H})) \text{ and}\end{aligned}$$

$$\begin{aligned}(Mat\,(\tilde{F}) \odot Mat\,(\tilde{G})) \odot Mat\,(\tilde{H}) &= ([f_{ij}] \odot [g_{ij}]) \odot [h_{ij}] \\ &= [f_{ij}] \odot ([g_{ij}] \odot [h_{ij}]) \\ &= Mat\,(\tilde{F}) \odot (Mat\,(\tilde{G}) \odot Mat\,(\tilde{H})).\end{aligned}$$

This proves (iii).

$Mat\,(\tilde{F}) \oplus Mat\,(\tilde{\emptyset}) = [f_{ij}] \oplus [0_{ij}] = [f_{ij}] = [0_{ij}] \oplus [f_{ij}] = Mat\,(\tilde{\emptyset}) \oplus Mat\,(\tilde{F})$,

$Mat\,(\tilde{F}) \odot Mat\,(\tilde{\emptyset}) = [f_{ij}] \odot [0_{ij}] = [0_{ij}] = [0_{ij}] \odot [f_{ij}] = Mat\,(\tilde{\emptyset}) \odot Mat\,(\tilde{F})$,

$$\begin{aligned}Mat\,(\tilde{F}) \oplus Mat\,(\tilde{X}) &= [f_{ij}] \oplus [1_{ij}] = [f_{ij} \oplus 1_{ij}] = [1_{ij}] = [1_{ij} \oplus f_{ij}] = [1_{ij}] \oplus [f_{ij}] \\ &= Mat\,(\tilde{X}) \oplus Mat\,(\tilde{F}),\end{aligned}$$

$Mat\,(\tilde{F}) \odot Mat\,(\tilde{X}) = [f_{ij}] \odot [1_{ij}] = [f_{ij}] = [1_{ij}] \odot [f_{ij}] = Mat\,(\tilde{X}) \odot Mat\,(\tilde{F})$.

This proves (iv).

$$\begin{aligned}Mat\,(\tilde{F}) \oplus (Mat\,(\tilde{F}) \odot Mat\,(\tilde{G})) &= [f_{ij}] \oplus ([f_{ij}] \odot [g_{ij}]) = [f_{ij} \oplus (f_{ij} \odot g_{ij})] \\ &= [f_{ij}] = Mat\,(\tilde{F})\end{aligned}$$

$$\begin{aligned}Mat\,(\tilde{F}) \odot (Mat\,(\tilde{F}) \oplus Mat\,(\tilde{G})) &= [f_{ij}] \odot ([f_{ij}] \oplus [g_{ij}]) = [f_{ij} \odot (f_{ij} \oplus g_{ij})] \\ &= [f_{ij}] = Mat\,(\tilde{F}), \text{ This proves (v)}\end{aligned}$$

$$\begin{aligned}Mat\,(\tilde{F}) \oplus (Mat\,(\tilde{G}) \odot Mat\,(\tilde{H})) &= [f_{ij}] \oplus ([g_{ij}] \odot [h_{ij}]) \\ &= ([f_{ij}] \oplus [g_{ij}]) \odot ([f_{ij}] \oplus [h_{ij}]) \\ &= (Mat\,(\tilde{F}) \oplus (Mat\,(\tilde{G})) \odot (Mat\,(\tilde{F}) \oplus Mat\,(\tilde{H}))\end{aligned}$$

$Mat\,(\tilde{F}) \odot (Mat\,(\tilde{G}) \oplus Mat\,(\tilde{H})) = [f_{ij}] \odot ([g_{ij}] \oplus [h_{ij}])$

$= ([f_{ij}] \odot [g_{ij}]) \oplus ([f_{ij}] \odot [h_{ij}])$

$= (Mat\,(\tilde{F}) \odot (Mat\,(\tilde{G})) \oplus (Mat\,(\tilde{F}) \odot Mat\,(\tilde{H}))$.

This proves (vi).

$Mat\,(\tilde{F}) \oplus (Mat\,(\tilde{F}))' = [f_{ij}] \oplus [f_{ij}]' = [1_{ij}] = [f_{ij}]' \oplus [f_{ij}]$

$= (Mat\,(\tilde{F}))^* \oplus Mat\,(\tilde{F})$

$Mat\,(\tilde{F}) \odot (Mat\,(\tilde{F}))' = [f_{ij}] \odot [f_{ij}]' = [0_{ij}] = [f_{ij}]' \odot [f_{ij}]$

$= (Mat\,(\tilde{F}))' \odot Mat\,(\tilde{F})$.

This proves (vii). This completes the proof of the theorem.

Theorem 2.5:

The algebraic structure $(M(\tilde{S}(X, E)), \oplus, \odot, ', [0_{ij}], [1_{ij}])$ is a Boolean algebra.

Proof: Follows from Theorem 2.4.

Conclusion

In this study, a matrix-based representation of soft sets over finite universes and parameter spaces has been developed. Each soft set is represented by a unique binary matrix, providing a simple and systematic way to encode parameterized information. The correspondence between soft sets and binary matrices was established as a bijection, and the basic soft set operations were expressed through entrywise matrix operations. The principal algebraic laws governing these operations were also verified, including idempotent, commutative, associative, absorption, distributive, identity, and complementation properties. Consequently, the resulting matrix structure was shown to form a Boolean algebra. This formulation offers a convenient algebraic and computational representation of finite soft sets and can serve as a foundation for further studies involving matrix-based soft set models and their applications.

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