

VISCOELASTIC FREE CONVECTIVE HEAT AND MASS TRANSFER PAST A VERTICAL CONE WITH THERMAL RADIATION EFFECTS

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Abstract

A numerical solution of transient laminar free convective heat and mass transfer in a viscoelastic fluid past a vertical cone in non-Darcian porous media in the presence of thermal radiation is presented. The Walters-B liquid model is employed to simulate medical creams and other rheological liquids encountered in biotechnology and chemical engineering. This rheological model introduces supplementary terms into the momentum conservation equation. The dimensionless governing equations of the flow are solved by an implicit finite difference scheme of Crank-Nicolson type. The velocity, temperature and concentration fields have been studied for the effect of radiation parameter, viscoelasticity parameter, Prandtl number, Schmidt number, buoyancy ratio parameter, Darcy number, Grashof number, Forchheimer number and semi vertical angle. The local skin friction, Nusselt number and Sherwood number are also presented and analyzed graphically. The numerical results are validated by comparisons with previously published work and are found to be in excellent agreement.

Key words: cone; viscoelasticity; non-Darcian porous medium; finite difference method;

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1. NOMENCLATURE

r	local radius of cone
x, y	coordinates along the cone generator and normal to the generator respectively
u, v	velocity components along the x - and y - directions respectively
g	gravitational acceleration
Gr_L	Grashof number
t'	time
t	dimensionless time
Nu_x	local Nusselt number
Nu_X	non-dimensional local Nusselt number
Pr	Prandtl number
R	dimensionless local radius of the cone
q_r	radiative heat flux
T'	temperature
T	dimensionless temperature
C'	concentration
C	dimensionless concentration
Da	Darcy number
Fs	Forchheimer inertial drag parameter
D	mass diffusion coefficient
U, V	dimensionless velocity components along the X - and Y - directions respectively
X, Y	dimensionless spatial coordinates along the cone generator

	and normal to the generator respectively
L	reference length
Sc	Schmidt number
k_0	Walters-B viscoelasticity parameter
Γ	dimensionless Walters-B viscoelasticity parameter
N	buoyancy ratio parameter
F	radiation parameter
Sh_x	local Sherwood number
Sh_X	dimensionless local Sherwood number
b	Forchheimer geometrical constant

Greek symbols

α	thermal diffusivity
β	volumetric thermal expansion coefficient
β^*	volumetric concentration expansion coefficient
ϕ	semi vertical angle of the cone
μ	dynamic viscosity
ν	kinematic viscosity
Δt	dimensionless time-step
ρ	density
ΔX	dimensionless finite difference grid size in X-direction
ΔY	dimensionless finite difference grid size in Y-direction
τ_x	local skin friction
τ_X	dimensionless local skin-friction

Subscripts

w	condition on the wall
∞	free stream condition

II. INTRODUCTION

In recent years, the problem of radiation effect on convection boundary layer flow has caused much attention because of the important applications in space technology and processes involving high temperatures. Nuclear power plants, gas turbines and the various propulsion devices for air craft, missiles and space vehicles are examples of such engineering areas. Several studies have examined theoretically the effect of thermal radiation on viscoelastic convection flows. The Rosseland diffusion flux model and a short-memory (Walters-B) non-Newtonian model was used in [12] to study the effect of elastic parameter and thermal radiation on viscoelastic flow. The effect of radiation on natural convection about an isothermal truncated cone was investigated in [16]. Viscoelastic flows and transport phenomena arise in numerous areas of chemical, industrial process, biosystems, food processing and biomedical engineering. These include the rheology of liquid crystal precursors employed in the manufacture of carbon super-fibers, crude oil emulsion processing, paper coating rheological processing, propulsive ciliary transport of respiratory airway mucus, thermocapillary bubble dynamics in weakly elastic fluids, rheo-reactor phosphatation flows, flour rheology, mayonnaise elastico-viscous flows, xanthan gum hydrogel flows, polygalacturonase-based food stuff.

Darcy's Law has been used exclusively in studies of porous-medium flow in reservoirs, however, there is considerable evidence that high-velocity non-Darcy flow occurs in many subsurface systems, such as in the flow near wells of oil or gas production, groundwater pumping, and liquid waste injection. Darcy's law, describing a linear relationship between volumetric flow rate (Darcy velocity) and pressure (or potential) gradient, has been the fundamental principle in analyzing flow processes in porous media. There are a few studies about the natural convection from cones in porous media saturated with Newtonian and non-Newtonian fluids. Reference [4] studied the problem of natural convection of a Darcian fluid about a cone. The coupled heat and mass transfer by free convection over truncated cone in porous media for variable wall temperature and concentration as well as for variable heat flux and variable mass flux was examined in [15]. Reference [5] used integral method to study heat and mass transfer by natural convection from truncated cones in porous media with variable wall temperature and concentration. The problem of natural convection of a non-Newtonian fluid from a vertical truncated cone was studied in [6] with variable wall temperature and concentration. In analyzing non-Darcy flow through porous media, the *Forchheimer* equation has been exclusively used to describe non-Darcy porous media flow, and has been extended to multiphase flow conditions. Recent laboratory studies and analyses have shown that the Barree and Conway model [2] is able to describe the entire range of relationships between flow rate and potential gradient from low to high flow rates through porous media, including those in transitional zones. Overall, this work provides an improved platform for modeling multiphase non-Darcy flow in oil and gas reservoirs, including complex fractured systems such as shale gas reservoirs.

The above studies did not consider combined viscoelastic momentum, heat and mass transfer from a vertical cone, either for the steady case or unsteady case in the presence of thermal radiation. Owing to the significance of this problem in chemical technological processing the *transient* case of such a flow is considered in this paper.

III. Constitutive equations for the Walters-B Viscoelastic Model

$$p_{ik} = -pg_{ik} + p_{ik}^* \quad (1)$$

$$p_{ik}^* = 2 \int_{-\infty}^t \Psi(t-t^*) e_{ik}^{(1)}(t^*) dt^* \quad (2)$$

$$\Psi(t-t^*) = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{-(t-t^*)/\tau} d\tau \quad (3)$$

where p_{ik} is the stress tensor, p is arbitrary isotropic pressure, g_{ik} is the metric tensor of a fixed coordinate system x_i , $e_{ik}^{(1)}$ is the rate of strain tensor and $N(\tau)$ is the distribution function of relaxation times, τ . The following generalized form of (2) has been shown in [14] to be valid for all classes of motion and stress.

$$p^{*ik}(x, t) = 2 \int_{-\infty}^t \Psi(t-t^*) \frac{\partial x^i}{\partial x^{*m}} \frac{\partial x^k}{\partial x^{*r}} e^{(1)mr}(x^* t^*) dt^* \quad (4)$$

in which $x_i^* = x_i^*(x, t, t^*)$ denotes the position at time t^* of the element which is instantaneously at the position, x_i , at time, t . Liquids obeying the relations (1) and (4) are

of the Walters-B' type. For such fluids with short memory i.e. low relaxation times, equation (4) may be simplified to:

$$p^{*ik}(x,t) = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\partial e^{(1)ik}}{\partial t} \quad (5)$$

in which $\eta_0 = \int_0^\infty N(\tau) d\tau$ defines the limiting Walters-B' viscosity at low shear rates,

$k_0 = \int_0^\infty \tau N(\tau) d\tau$ is the Walters-B' viscoelasticity parameter and $\frac{\partial}{\partial t}$ is the convected time

derivative. This rheological model is very versatile and robust and provides a relatively simple mathematical formulation which is easily incorporated into boundary layer theory for engineering applications

IV. MATHEMATICAL FORMULATION

An axi-symmetric unsteady laminar free convection heat and mass transfer in a viscoelastic, incompressible, absorbing-emitting, non-scattering, optically-thick fluid from an isothermal vertical cone with uniform surface temperature and concentration in a Darcy-Forchheimer fluid saturated isotropic porous medium in the presence of thermal radiation is considered. Unsteady incompressible flow and non-reactive mass diffusion in a free convective viscoelastic flow driven by species buoyancy force occurs upwards along the cone. In addition, the formulation of mathematical equations is based on the following assumptions:

The physical properties of the fluid are constant except density in the buoyancy force term in the momentum equation.

The density is a linear function of temperature and species concentration given by $\rho = \rho_0 [1 - \beta^* \{(T' - T'_0) + \beta(C' - C'_0)\}]$. Hence, the Boussinesq's approximation is taken into account for the buoyancy effects induced by thermal and mass diffusion. Viscous dissipation effects and pressure gradient along the boundary layer are negligible. The magnetic Reynolds number is very small so that the induced magnetic field is negligible. The magnetic field is not strong enough to cause Joule heating so that the term due to electrical dissipation is neglected in energy equation. The concentration C' of the diffusing species in the binary mixture is assumed to be very less in comparison to the other chemical species, which are present. This leads to the assumption that the Soret and Dufour effects are negligible. The chemical reaction in the fluid is also negligible. Also implicit in our analysis is the assumption that the cone surface and the surrounding fluid which are at rest possess the same temperature T'_∞ and concentration level C'_∞ . At time $t' > 0$, the temperature of the cone surface and the concentration level near the cone surface are elevated and are sustained as constant thereafter. The co-ordinate system chosen (as shown in Fig.1) is such that the x -coordinate is directed along the surface of the cone from the apex ($x = 0$) and the y -coordinate is orientated perpendicular to this i.e. at right angles to the cone surface, outwards. Here, ϕ designates the semi-vertical angle of the cone and r is the local radius of the cone.

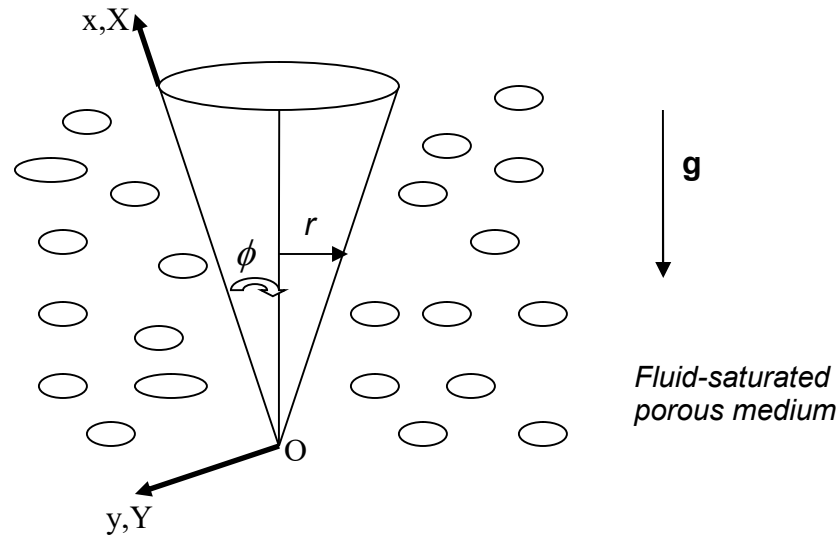


Figure 1: Physical Model

Under the above assumptions, implementing the shear-stress strain tensor for a Walters-B liquid, the appropriate unsteady incompressible conservation equations for the regime may be shown to take the form:

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0 \quad (6)$$

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \frac{\partial^3 u}{\partial y^2 \partial t'} + g\beta \cos \phi (T' - T'_\infty) + g\beta^* \cos \phi (C' - C'_\infty) - \frac{\nu}{K} u - \frac{b}{K} u^2 \quad (7)$$

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (8)$$

$$\frac{\partial C'}{\partial t'} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} \quad (9)$$

The initial and boundary conditions are prescribed as:

$$\begin{aligned} t' \leq 0 : u = 0, v = 0, T' = T'_\infty, C' = C'_\infty & \quad \text{for all } x, y, \\ t' > 0 : u = 0, v = 0, T' = T'_w, C' = C'_w & \quad \text{at } y = 0, \\ u = 0, T' = T'_\infty, C' = C'_\infty & \quad \text{at } x = 0, \\ u \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty & \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (10)$$

Where all parameters are defined in the nomenclature. In the momentum (velocity boundary layer) Equation (7) the second term on the right hand side is the viscoelastic mixed derivative term, the third term is thermal buoyancy force, the fourth term is the species (concentration) buoyancy force, the penultimate term is the Darcian linear drag and the final term is the Forchheimer quadratic drag force. Now we implement the Rosseland approximation [12]. Refractive index of the fluid medium is constant.

Unidirectional radiation flux, q_r , is considered and it is assumed that $\frac{\partial q_r}{\partial y} \ll \frac{\partial q_r}{\partial x}$. This

model is valid for *optically-thick* media in which thermal radiation propagates only a limited distance prior to experiencing scattering or absorption. The local thermal

radiation intensity is due to radiation emanating from proximate locations in the vicinity of which emission and scattering are comparable to the location of interest. For zones where conditions are appreciably different, thermal radiation has been shown to be greatly attenuated before arriving at the location under consideration. The energy transfer depends on conditions only in the area adjacent to the plate regime i.e. the boundary layer regime. Rosseland's model is physically valid for intensive absorption i.e. *optically-thick flows* which are optically far from the bounding surface. Implicit in this approximation is also the existence of wavelength regions where the optical thickness may exceed a value of five. Under this approximation we arrive at the following expression for uni-directional radiative heat flux q_r :

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T'^4}{\partial y} \quad (11)$$

where σ_s is the Stefan-Boltzmann constant and k_e is the mean absorption coefficient, respectively. The optically-thick assumption is realistic for certain polymeric fluids [13]. If temperature differences within the flow are sufficiently small, then Equation (11) can be linearized by expanding T'^4 into the Taylor series about T'_∞ , which after neglecting higher order terms takes the form:

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \quad (12)$$

In view of Equations (11) and (12), Equation (8) reduces to

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} + \frac{16\sigma_s T_\infty'^3}{3k_e \rho c_p} \frac{\partial^2 T'}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (13)$$

where all parameters are defined in the nomenclature.

Local skin-friction (wall shear stress), local Nusselt number and local Sherwood number are given respectively by:

$$\tau_x = -\mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (14)$$

$$Nu_x = \frac{-x \left(\frac{\partial T'}{\partial y} \right)_{y=0}}{T'_w - T'_\infty} \quad (15)$$

$$Sh_x = \frac{-x \left(\frac{\partial C'}{\partial y} \right)_{y=0}}{C'_w - C'_\infty} \quad (16)$$

Proceeding with analysis, we implement the following non-dimensional quantities to facilitate a numerical solution to the boundary value problem defined by Equations (6) to (9) under conditions (10):

$$\begin{aligned} X &= \frac{x}{L}, & Y &= \frac{y}{L} (Gr_L)^{\frac{1}{4}}, & R &= \frac{r}{L}, & \text{where } r &= x \sin \phi, \\ V &= \frac{\nu L}{\nu} (Gr_L)^{-\frac{1}{4}}, & U &= \frac{uL}{\nu} (Gr_L)^{-\frac{1}{2}}, & t &= \frac{\nu t'}{L^2} (Gr_L)^{\frac{1}{2}}, & Da &= \frac{K}{L^2} \end{aligned} \quad (17)$$

$$T = \frac{T' - T'_\infty}{T'_w(L) - T'_\infty}, \quad Gr_L = \frac{g\beta(T'_w(L) - T'_\infty)L^3}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}, \quad F = \frac{k_e k}{4\sigma_s T_\infty^3}$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad \Gamma = \frac{k_0 Gr_L^{\frac{1}{2}}}{L^2}, \quad N = \frac{\beta^*(C'_w - C'_\infty)}{\beta(T'_w - T'_\infty)}, \quad Sc = \frac{\nu}{D}, \quad Fs = \frac{b}{L}$$

Equations (6), (7), (13) and (9) are reduced to the following non-dimensional form

$$\frac{\partial(UR)}{\partial X} + \frac{\partial(VR)}{\partial Y} = 0 \quad (18)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - \Gamma \frac{\partial^3 U}{\partial Y^2 \partial t} + T \cos \phi + NC \cos \phi - \frac{U}{Da Gr_L} - \frac{Fs}{Da} U^2 \quad (19)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \left[1 + \frac{4}{3F} \right] \frac{\partial^2 T}{\partial Y^2} \quad (20)$$

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \quad (21)$$

The corresponding initial and boundary conditions are

$$\begin{aligned} t \leq 0 : U &= 0, V = 0, T = 0, C = 0 && \text{for all } X, Y, \\ t > 0 : U &= 0, V = 0, T = 1, C = 1 && \text{at } Y = 0, \\ U &= 0, T = 0, C = 0 && \text{at } X = 0, \\ U &\rightarrow 0, T \rightarrow 0, C \rightarrow 0 && \text{as } Y \rightarrow \infty. \end{aligned} \quad (22)$$

where again all parameters are provided in the nomenclature. The dimensionless local values of the skin friction (surface shear stress), the Nusselt number (surface heat transfer gradient) and the Sherwood number (surface concentration gradient) are given by the following expressions:

$$\tau_x = - \left(\frac{\partial U}{\partial Y} \right)_{Y=0} \quad (23)$$

$$Nu_x = -X \left(\frac{\partial T}{\partial Y} \right)_{Y=0} \quad (24)$$

$$Sh_x = -X \left(\frac{\partial C}{\partial Y} \right)_{Y=0} \quad (25)$$

V. NUMERICAL SOLUTION

In order to solve the unsteady, non-linear, coupled equations (18)–(21) under the conditions (22), an implicit finite difference scheme of Crank-Nicolson type which is discussed in [1], [7], [9] and [10]. The finite difference scheme of dimensionless governing equations is reduced to tri-diagonal system of equations and is solved by Thomas algorithm as discussed in [3]. The region of integration is considered as a rectangle with $X_{\max} = 1$ and $Y_{\max} = 22$ where $Y_{\max}$ corresponds to $Y = \infty$ which lies very well out side both the momentum and thermal boundary layers. The maximum of Y was chosen as 22, after some preliminary investigation so that the last two boundary

conditions of (16) are satisfied within the tolerance limit 10^{-5} . The mesh sizes have been fixed as $\Delta X = 0.05, \Delta Y = 0.05$ with time step $\Delta t = 0.01$. The computations are carried out first by reducing the spatial mesh sizes by 50% in one direction, and later in both directions by 50%. The results are compared. It is observed in all cases, that the results differ only in the fifth decimal place. Hence, the choice of the mesh sizes seems to be appropriate. The scheme is unconditionally stable. The local truncation error is $O(\Delta t^2 + \Delta Y^2 + \Delta X)$ and it tends to zero as $\Delta t, \Delta X$ and ΔY tend to zero. Hence, the scheme is compatible. Stability and compatibility ensure the convergence.

VI. RESULTS AND DISCUSSION

Only selective figures have been reproduced here for brevity. Default values of the thermophysical parameters are specified as follows: radiation parameter (F) = 3.0 (strong thermal radiation compared with thermal conduction), viscoelasticity parameter (Γ) = 0.005, buoyancy ratio parameter (N) = 1.0, Schmidt number (Sc) = 0.6 (oxygen diffusing in the viscoelastic fluid), Prandtl number (Pr) = 0.7 (water-based solvents), semi-vertical angle of the cone (ϕ) = 20° , Grashof number (Gr_L) = 1.0, Darcy number (Da) = 0.1 (high permeability), Forchheimer number (Fs) = 0.1 (weak quadratic drag). All graphs therefore correspond to these values unless otherwise indicated. In order to prove the accuracy of the computations in steady state at $X = 1.0$, $Pr = 0.7$, $\eta = Y$ and considering

$Gr_L^* = Gr_L \cos \phi = \frac{g \beta \cos \phi (T_w' - T_\infty') L^3}{\nu^2}$ are compared with available similarity solutions in

the literature. The local skin-friction τ_x and local Nusselt number Nu_x values for different Prandtl number are compared with the results of isothermal case ($n = 0$) of [8] and [11] in Tables 1, 2 respectively, and are found to be in good agreement.

Table 1 Comparison of steady state local skin-friction values at $X = 1.0$ with those of [8] and [11]

Pr	Hering[8]		Pullepu Bapuji et al.[11] $\tau_x / Gr_L^{3/4}$	Present Results
	$f''(0)$	$f''(0)\sqrt{Pr}$		
0.03	7.185	1.2444	1.2406	1.24270
0.1	3.466	1.0960	1.0932	1.09525
0.7	0.9796	0.8195	0.8163	0.81312
1.0	0.7694	0.7694	0.7660	0.76665

Table 2 Comparison of steady state local Nusselt number values at $X = 1.0$ with those of [8] and [11]

Pr	Hering[8]		Pullepu Bapuji et al.[11] $Nu_x / Gr^{1/4}$	Present Results
	$-\theta'(0)$	$-\theta'(0)\sqrt{\text{Pr}}$		
0.03	0.7185	0.12440	0.1258	0.12524
0.1	0.6683	0.2113	0.2121	0.21202
0.7	0.5392	0.4511	0.4554	0.45254
1.0	0.5104	0.5104	0.5157	0.51461

Figures 1(a) and 1(b) show the influence of radiation parameter, F , on steady state velocity (U) and temperature (T) distributions with distance into the boundary layer, transverse to the cone surface (Y). $F = \frac{k_e k}{4\sigma_s T_\infty^3}$ and this defines the ratio of thermal

conduction contribution relative to thermal radiation. For radiative heat transfer dominance in the boundary layer regime, $F \rightarrow 0$. For finite values of F there will be a simultaneous presence of thermal conduction and radiative transfer contributions. For $F = 1$, both modes will contribute equally. An increase in F from 0 (total thermal radiation dominance) through 0.5, 1.0, 3.0, 5.0, 10.0 to 100.0, causes a significant reduction in velocity with distance into the boundary layer i.e. retards the flow. With an increase in F (decrease in thermal radiation contribution) the *time* taken to attain the steady state is markedly lowered. Increasing thermal radiation flux i.e. reduction in F , therefore delays the time taken to attain the steady state in the boundary layer regime. Temperature values (T) are also considerably reduced with an increase in F as there is a progressive decrease in thermal radiation contribution accompanying this. **Figure 2** depict the response of the steady state temperature (T) distributions to different values of the viscoelastic material parameter, Γ . An increase in Γ from 0 to 0.001, 0.003 and to the largest value of 0.005, decrease the temperature distributions accompanies an increase in Γ . In **Figure 3**, the distributions of steady-state streamwise temperature function (T) versus spanwise coordinate (Y) for various buoyancy ratio parameters (N) close to the leading edge (i.e.

cone apex) at $X = 1.0$, are shown. The parameter $N = \frac{\beta^*(C'_w - C'_\infty)}{\beta(T'_w - T'_\infty)}$ defines the ratio of

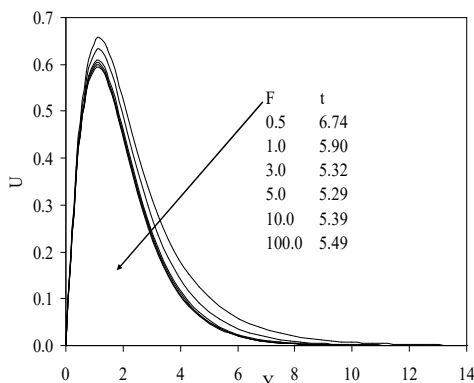
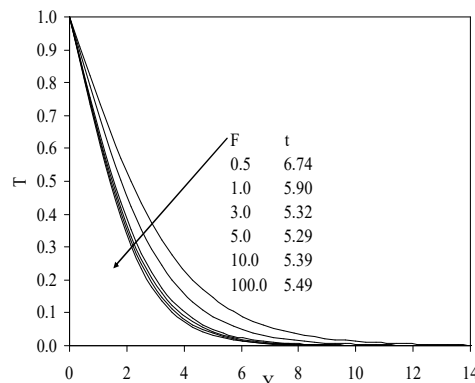
the species (mass diffusion) buoyancy force to the thermal (heat diffusion) buoyancy force. An increase in N from 0 through 1, 2, 3, 4 to 5 clearly decreases temperature (T) throughout the boundary layer transverse to the cone surface and that again increasing N is responsible for a decrease in the time taken to attain the steady-state. **Figure 4** depict the distribution of streamwise concentration (C) versus spanwise coordinate (Y) for various Schmidt numbers (Sc). An increase in Sc from 0.1 (low weight diffusing gas

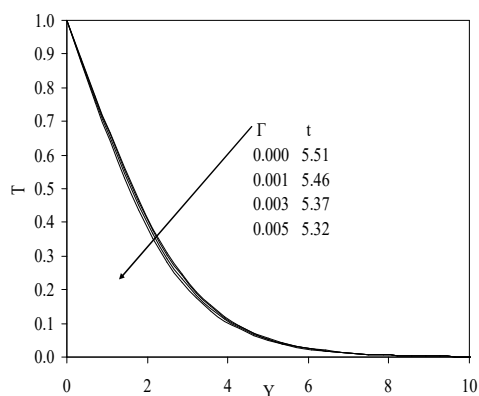
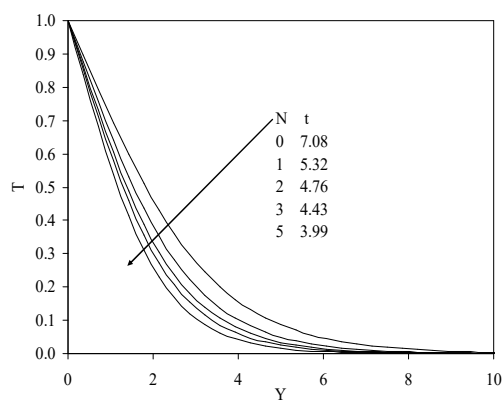
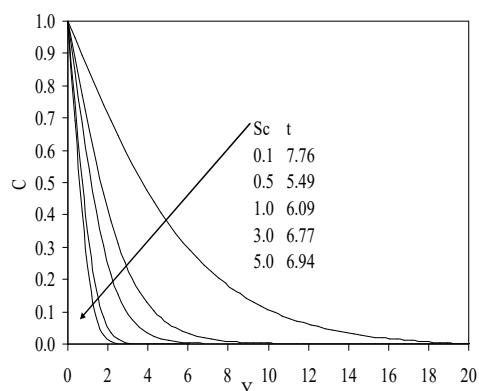
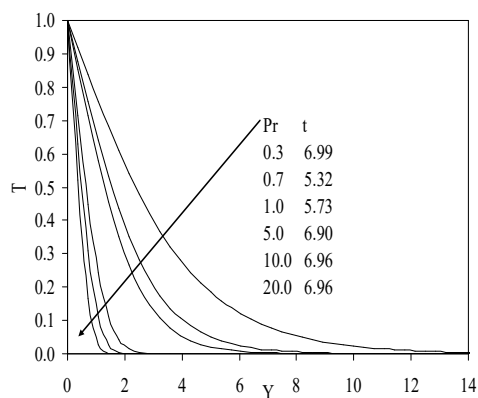
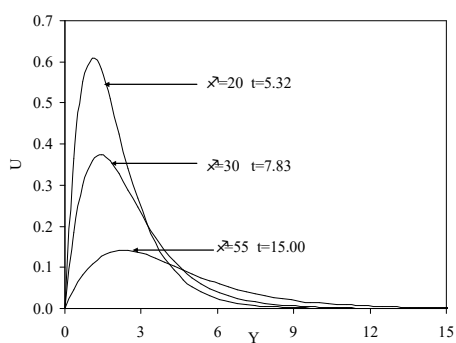
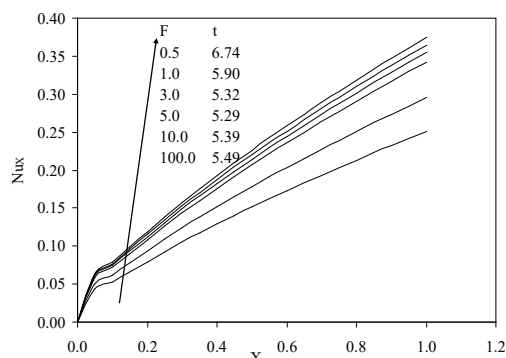
species) through 0.5 (oxygen diffusing) to 1.0 (denser hydrocarbon derivatives as the diffusing species), 3.0 and 5.0, clearly strongly *suppresses* concentration levels in the boundary layer regime. An increase in *Schmidt number effectively* depresses concentration values in the boundary layer regime since higher Sc values will physically manifest in a decrease of molecular diffusivity (D) of the viscoelastic fluid i.e. a reduction in the rate of mass diffusion. **Figure 5** illustrate the spanwise spatial response of the flow variables to Prandtl number (Pr). Pr defines the ratio of momentum diffusivity to thermal diffusivity. $Pr < 1$ physically corresponds to cases where heat diffuses faster than momentum. $Pr = 0.7$ is representative of water-based solvents and $Pr \gg 1$ e.g. 10, 20 corresponds to lubricating oils, polymers etc. With increasing Pr from 0.3 through 0.7, 1.0, 5.0, 10.0 to 20.0, temperature, T as shown in figure, is markedly reduced throughout the boundary layer. Time taken to arrive at the steady state profiles for temperature, initially decrease with a rise in Prandtl number from 0.3 up to 1; thereafter however the trend is reversed and time elapsing to the steady state is increased with increasing Prandtl number from 1 through 5 to 10. **Figure 6** shows the spanwise spatial distributions of the velocity (U) for different values of semi-apex cone angle, ϕ . With wider cone angles i.e. a rise from 20 through 30 to 50 degrees, the velocity profiles are considerably *reduced* close to the cone surface i.e. the flow is strongly *decelerated* near the wall. **Figure 7** show the variation of dimensionless Nusselt number (surface heat transfer gradient), Nu_x versus streamwise coordinate (X) for various conduction-radiation parameters (F). Increasing F values strongly enhance the local Nusselt number, Nu_x values. Therefore with greater thermal radiation (lower F) Nu_x values are smaller i.e. heat transfer rates at the cone surface are suppressed with increasing thermal radiation. Again increasing F (progressively greater thermal *conduction* heat transfer contribution) causes a decrease in the time needed to attain the steady state. In **Figure 8** the variation of dimensionless local skin friction (surface shear stress), τ_x , versus streamwise coordinate (X) for various viscoelasticity parameters (Γ) are shown. Shear stress is strongly elevated with increasing viscoelasticity (i.e. stronger elastic effects) i.e. the flow is accelerated. In **Figure 9** the distributions of Sh_x , versus streamwise coordinate (X) for various Schmidt numbers (Sc) are depicted. With increasing Sc , local Sherwood number, Sh_x , is accentuated markedly along the cone surface. Again as described earlier, the time taken to attain the steady state is maximized for the lowest Sc value i.e. $t = 7.76$ for $Sc = 0.1$. In **Figure 10**, the influence of Grashof number (Gr_L) on steady state velocity (U) distributions with Y-coordinate are shown. It is observed that increasing Gr_L values from 0.1 through 1.0, 10.0, 50.0 and 100.0 clearly increases velocity. **Figure 11** shows the transient profiles of dimensionless temperature (T) distributions with Y-coordinate at $X = 1.0$ for various values of Darcy number (Da). It is observed that increasing Da values from 0.001 through 0.01, 0.1 and 1.0 reduces the temperature.

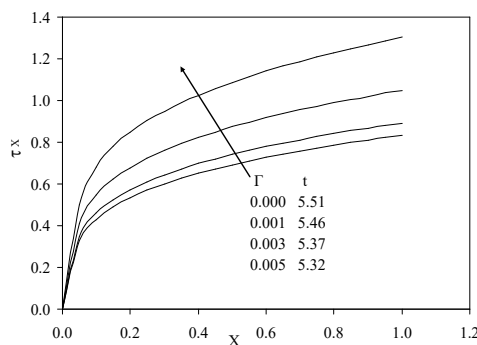
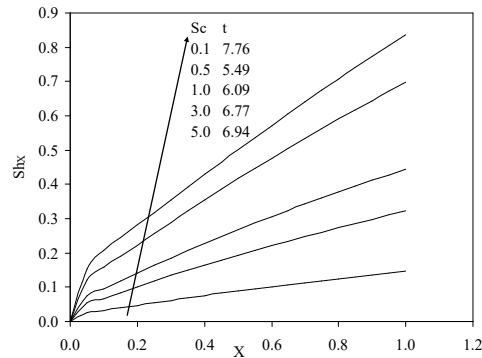
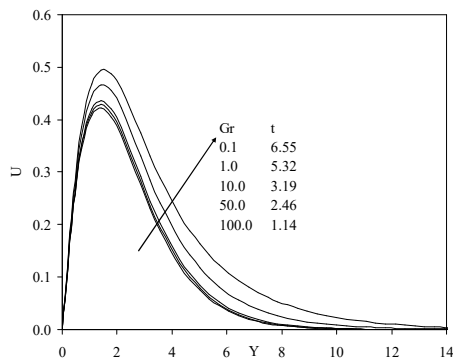
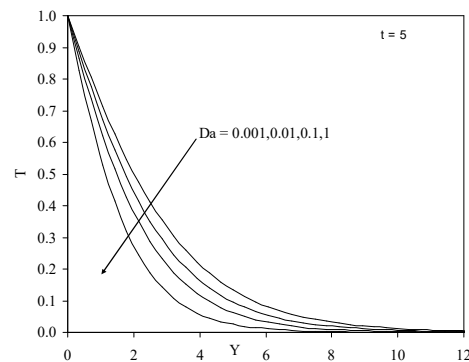
VII. CONCLUSIONS

Numerical finite difference solutions have been developed for the transient laminar buoyancy-driven boundary layer heat and mass transfer in a Walters-B viscoelastic flow from a vertical stationary cone with appreciable thermal radiation effects.

The computations have shown that increasing viscoelasticity (Γ) accelerates the streamwise velocity and shear stress (local skin friction), local Nusselt number and local Sherwood number. Increasing thermal radiation (decreasing F) accelerates the flow and increases temperatures in the boundary layer regime but reduces local Nusselt number at the cone surface and also suppresses concentration values. An increase in cone semi-apex angle (ϕ) is observed to markedly decelerate the flow near the cone surface but to increase temperatures and concentrations in the boundary layer regime. Increasing buoyancy ratio parameter (N) strongly accelerates the flow near the cone surface but suppresses temperature and concentration magnitudes. Increasing cone angle and thermal radiation also considerably increase the time taken to attain the steady state. Increasing the values of Darcian drag parameter (Da) or Grashof number (Gr_L) reduces the temperature and concentration but increase the velocity. It is observed that increasing the values of Forchheimer inertial drag parameter (F_s) leads to a fall in velocity but increases both the temperature and concentration. However increasing viscoelasticity and buoyancy ratio have a stabilizing effect on the transient regime and *decrease* the time required to attain the steady state. The present model has applications in high temperature polymeric and other chemical processing technologies.

Fig.1(a). Steady state velocity profiles at $X=1.0$ for different F Fig.1(b). Steady state temperature profiles at $X=1.0$ for different F

Fig.2.Steady state temperature profiles at X=1.0 for different Γ Fig.3.Steady state temperature profiles at X=1.0 for different N Fig.4.Steady state concentration profiles at X=1.0 for different Sc Fig.5. Steady state temperature profiles at X=1.0 for different Pr Fig.6. Steady state velocity profiles at X=1.0 for different λ Fig.7. Effect of F on local Nusselt number

Fig.8. Effect of Γ on local skin frictionFig.9. Effect of Sc on local Sherwood numberFig.10. Steady state velocity profiles at $X = 1.0$ for different Gr Fig.11. Transient temperature profiles at $X = 1.0$ for different Da

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